NOTE ON THE MECHANISM OF DECAY OF A JET INTO LARGE DROPS

Discussion of the article by M. I. Vivdenko and K. N. Shabalin (Journal of Engineering Physics, 8, No. 4, 1965)

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The article of Vivdenko and Shabalin gave an incongruous presentation of the present state of knowledge as regards the decay of a liquid jet, and proposed an erroneous mechanism (due to oscillations), without any real foundation, for this process, ascribing it to Rayleigh and claiming it to be widely applied at the present time. On this basis the authors presented a new decay mechanism taking account of the "basic property of a liquid—its fluidity." Again, Plateau [1] showed that if the surface of a cylinder of liquid of radius R_0 is deformed in such a way that it may be described by the equation (or by a system of equations of this type)

$$R = R_0 + \varepsilon \cos Kz, \qquad (1)$$

where R is the distance of the surface from the axis in the plane perpendicular to the axis, ε is a small quantity, K is the wave number, and z is the coordinate along the axis, then the deformation may be stable or unstable, depending on whether the product KR_0 is greater or less than unity. This is due to the fact that the surface of the deformed cylinder of length $\lambda (\lambda = 2\pi/K)$, greater than $2\pi R_0$, is less than the undisturbed surface. Because of the excitation of this kind of disturbance, part of the potential energy of the surface tension (proportional to the surface area) is transformed into kinetic energy of motion of the liquid, which is the cause of the irreversible decay of the jet.

The great merit of Rayleigh [2] was the development of this phenomenon with allowance for the dynamics of motion of the liquid, and the proof that disturbances with different lengths increase with different velocities, and it follows from this that the jet must decay into drops, formed from sections of the jet with wavelength of the disturbances that are increasing most rapidly.

Analyzing the increase of ε with time, Rayleigh showed that

$$\varepsilon = \varepsilon_0 \exp(qt), \qquad (2)$$

where ε_0 is the initial disturbance.

For axisymmetric disturbances

$$q^2 = \frac{T}{\rho R_0^3} \frac{KR_0 I_1(KR_0)}{I_0(KR_0)} [1 - (KR_0)^2], \qquad (3)$$

where ρ is the density, T the surface tension, and I_m the Bessel function of order m of imaginary argument.

It follows from (3) that $q^2 < 0$ for $KR_0 > 1$ and $q^2 > 0$ for $KR < 1$. In other words, the jet is unstable with regard to disturbances with wavelength greater than $2\pi R_0$.

This instability does not have an oscillatory character, but has the form of disturbances which grow with time, as an exponential function, at a given place. For $KR_0 > 1$ oscillations exist in the jet, but they are not the cause of its decay. Analysis of (3) shows that for $KR_0 = 0.697$, q^2 has a maximum. Therefore, the jet must break up into sections of length $2\pi R_0/0.697 = 4.51 \cdot 2R_0$.

Tyler [3] confirmed the experimental results of Rayleigh. Weber [4] extended the Rayleigh theory to viscous liquids. He also studied the interaction between the external flow and jet decay. The problem of the decay of a jet of liquid was again treated in the monographs of Levich [5] and Chandrasekhar [6]. The latter also worked on the problem of the stability of electrically conducting jets in a magnetic field. What Vivdenko and Shabalin represent as new is stated in all the existing theories. Even the second argument of the above authors that the source of the initial disturbances in the jet is "mainly the end of the jet, where drop breakaway occurs," fails to stand criticism.

For the end of the jet to be able to influence its decay, it is necessary that disturbances from the end be able to move upstream to the beginning of the jet.

The velocity of propagation of disturbances in the jet may be determined with the aid of the relation

$$V = \frac{q}{2\pi i} \lambda. \qquad (4)$$

Therefore, as a function of wavelength

$$V = \sqrt{\frac{I_1(2\pi R_0/\lambda)}{I_0(2\pi R_0/\lambda)} \frac{2\pi T}{\rho\lambda} \left(1 - \frac{\lambda^2}{4\pi^2 R_0^2}\right)}. \qquad (5)$$

It follows from this formula that only disturbances with wavelength $\lambda < 2\pi R_0$ may move along a cylindrical jet. Disturbances of this type, however, are not the cause of decay, as has already been shown. When $\lambda = 2\pi R_0$ (the limiting case of a stable jet), the velocity of propagation of disturbances in the jet is equal to zero, while when $\lambda > 2\pi R_0$ (decay of the jet), the velocity becomes imaginary. Thus, the latter case has nothing in common with waves in the full sense of the word, since the disturbances have no self-motion, and only move along with the jet like a wave. In other words, the disturbances causing decay cannot move from the end to the beginning of the system, and it is impossible to assert that the end of the jet has any kind of influence on its decay.

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